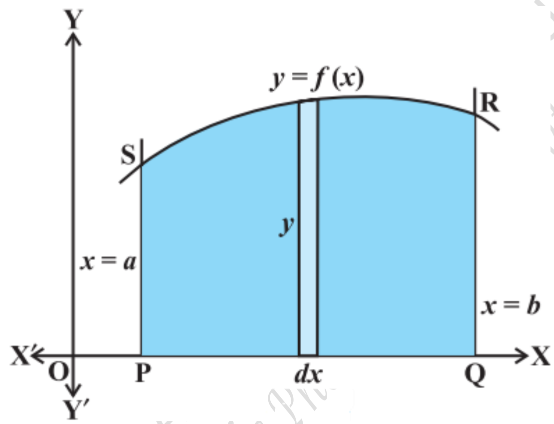
FORMULAE & KEY POINTS

CLASS 12 MATHEMATICS

CH. 8. APPLICATION OF INTEGRALS

1 AREA BOUNDED BY A GIVEN CURVE AND THE COORDINATE AXES

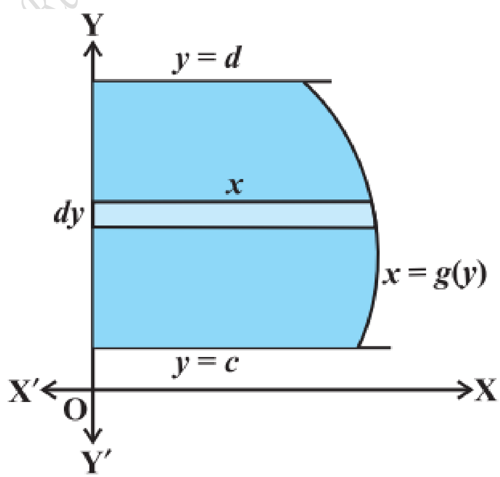
1.1



The area A of the region bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$ is give as

$$A = \int_a^b dA = \int_a^b y dx = \int_a^b f(x) dx$$

1.2

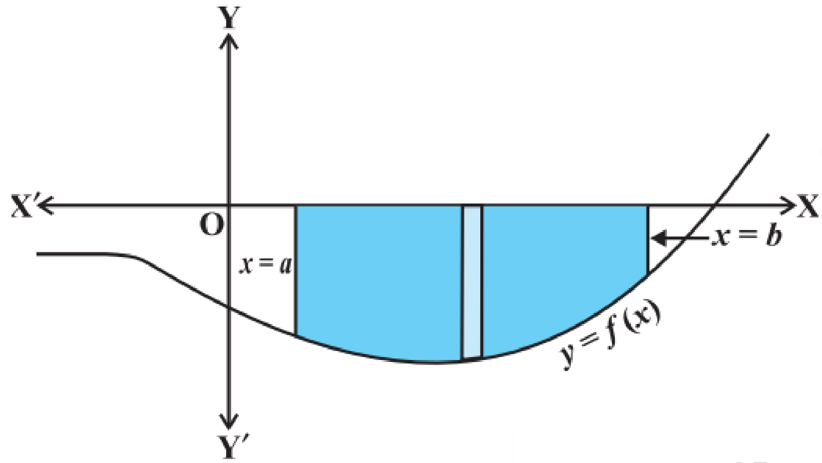


The area A of the region bounded by the curve $x = g(y)$, y-axis and the abscissae $y = c, y = d$ is given by

$$A = \int_c^d dA = \int_c^d x\,dy = \int_c^{bc} g(y) dy$$

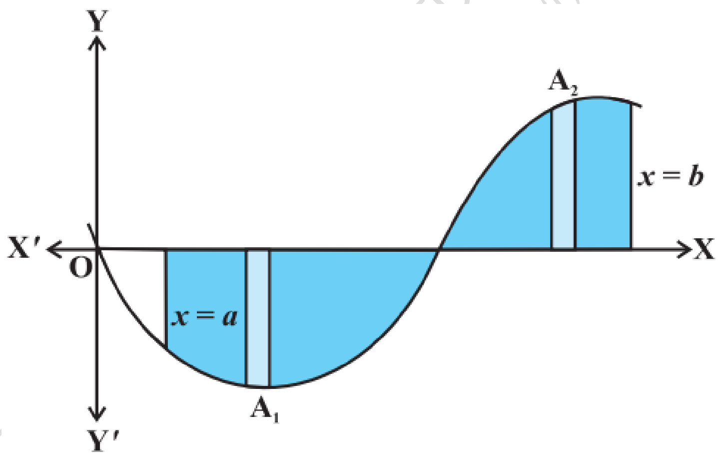
REMARKS

(i)



If the position of the curve under consideration is below the x-axis, then since $f\ (x) < 0$ from $x\ =\ a$ to $x\ =\ b$, the area bounded by the curve, x-axis and the ordinates $x =\ a$, $x\ =\ b$ comes out to be negative. But, it is only the numerical value of the area which is taken into consideration. Thus, if the area is negative, we take its absolute value, i.e., $\left|\int_a^b f(x)dx\right|$

(ii)



Generally, it may happen that some portion of the curve is above x-axis and some is below the x-axis as shown in the Fig. Here, $A_1\ <\ 0$ and $A_2\ >\ 0$. Therefore, the area A bounded by the curve $y\ =\ f\ (x), x\text{-}axis$ and the ordinates $x = a$ and $x =\ b$ is given by

$A\ =\ |A_1|\ +\ A_2$

(iii) To find the area of certain figures / regions which are divided by the coordinate axes into congruent parts having equal areas, we can find the required area A as

A = (number of congruent parts) × (area of a single part)

Thus, as shown in the figures given below,

(i) Area of circle $x^2 + y^2 = a^2$is given as

A = 4 × $\int_0^a x\ dy$ **OR** Area of circle = 4 × $\int_0^a y\ dx$ [see Fig.(i) and (ii)]

(ii) Area of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = a^2$ is given by

A = 4 × $\int_0^a x\ dy$ **OR** Area of ellipse = 4 × $\int_0^a y\ dx$ [see Fig.(iii) and (iv)]

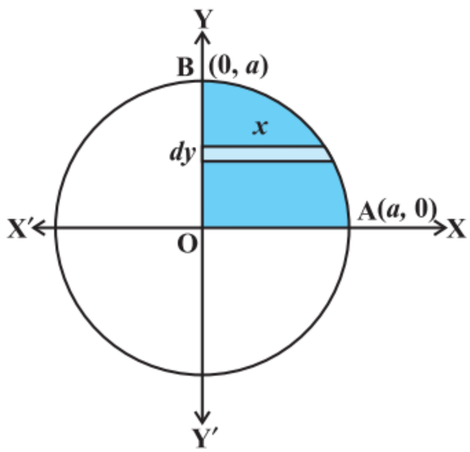


Fig.(i)

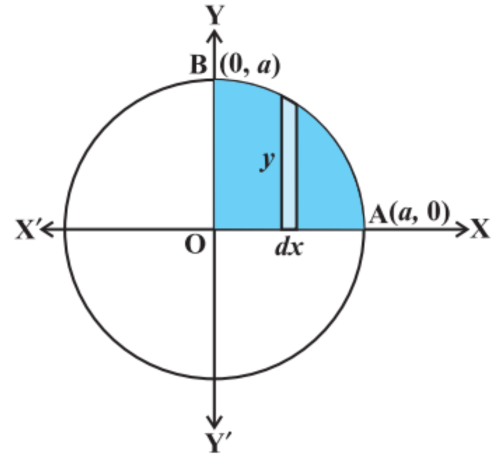


Fig.(ii)

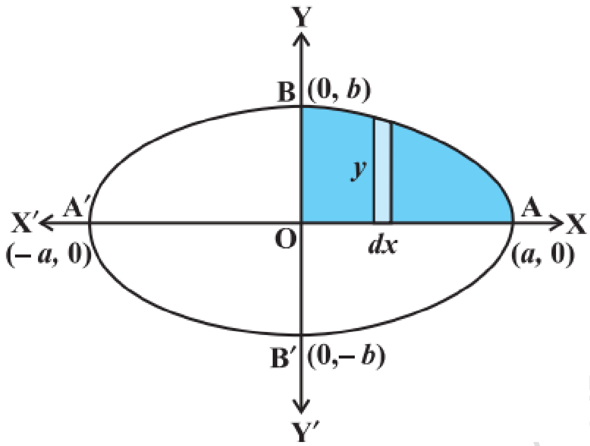


Fig.(iii)

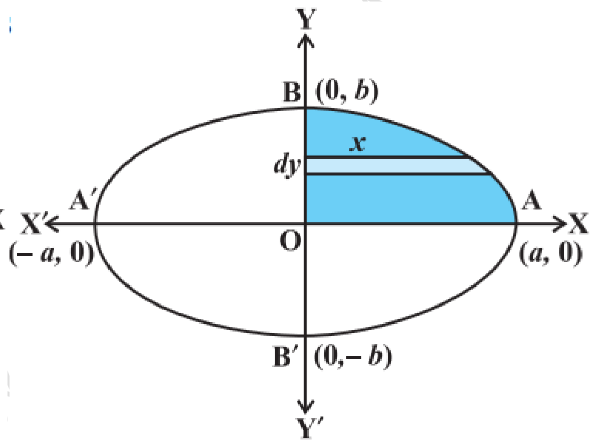


Fig.(iv)

(iii) Area A of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$ is given as

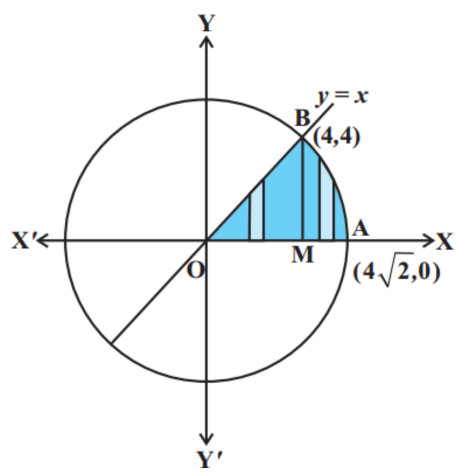
A = area of the region OBMO

= (Area of region OBMO) + (Area of region MABM)

$= \int_0^4 y_1\,dx + \int_4^{4\sqrt{2}} y_2\,dx$

where, $y_1 = y$ from the equation $y = x$,

$y_2 = y$ from the equation $x^2 + y^2 = 32$

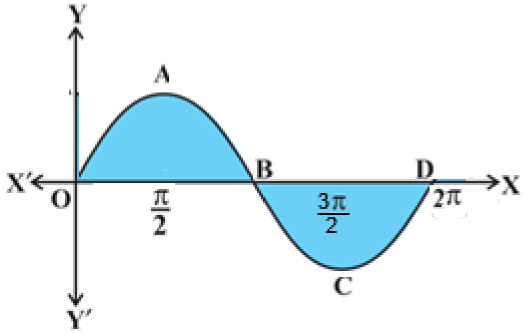


2. AREAS OF SOME IMPORTANT CURVES

2.1 Area of circle $x^2 + y^2 = r^2$= is πr^2

2.2 Area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = a^2$ is πab

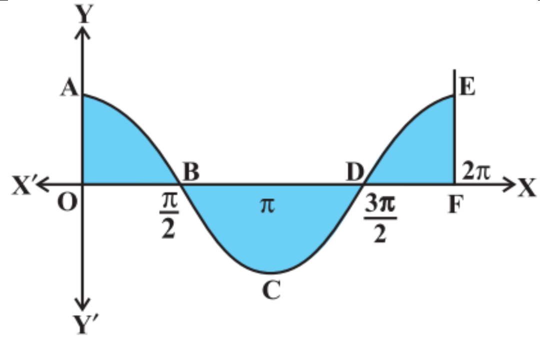
2.3



Area of function $y = \sin x$

$A = \int_0^{2\pi} \sin x \; dx \; = \; \int_0^{2\pi} \sin x \; dx$ =4units

2.4

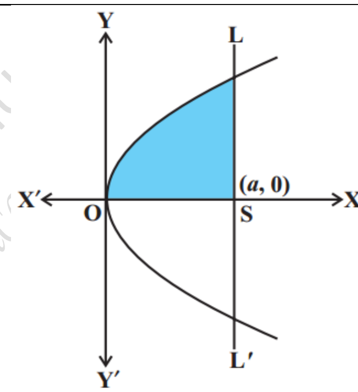


Area of function $y = \cos x$

$A = \int_0^{2\pi} \cos x \; dx \; = \; \int_0^{2\pi} \cos x \; dx$ = 4 units

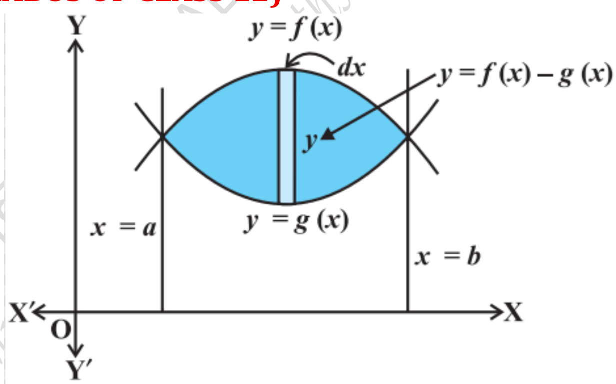
2.5 Area of the parabola $y^2 = 4ax$ bounded by its latus rectum $x = a$ is

$$A = \; 2 \times \left(\frac{4}{3}a^2\right) \text{sq. units} = \frac{8}{3}a^2 \text{ sq. units}$$



3. AREA BETWEEN TWO CURVES (NOT IN CBSE SYLLABUS OF CLASS 12)

3.1



Consider two curves represented by $y = f(x), y = g(x)$, where $f(x) \geq g(x)$ in $[a, b]$ as shown in Fig. Let the points of intersection of these two curves are given by $x = a$ and $x = b$.

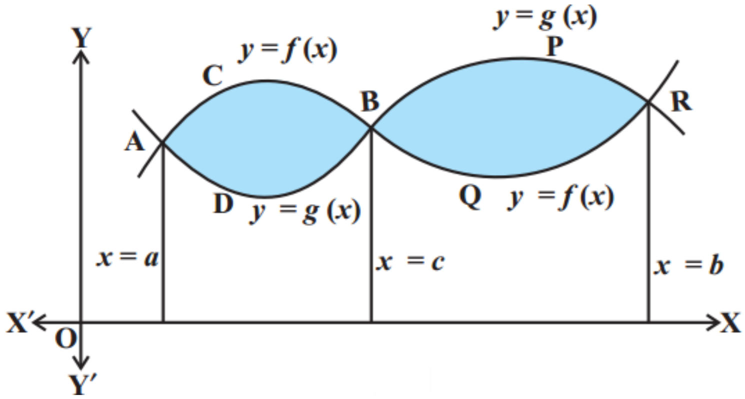
Then area A between the curves will be given as

A = [area bounded by $y = f(x), x -$ axis and the lines $x = a, x = b$]

$-$ [area bounded by $y = f(g), x -$ axis and the lines $x = a, x = b$]

$$= \int_a^b [f(x) - g(x)] \, dx$$

3.2



If $f(x) \geq g(x)$ in $[a, c]$ and $f(x) \leq g(x)$ in $[c, b]$, where $a < c < b$ as shown in the Fig. then the area A of the regions bounded by curves can be written as

A = (Area of the region ACBDA) + (Area of the region BPRQB)

$$= \int_a^b [f(x) - g(x)]\, dx + \int_c^d [g(x) - f(x)]\, dx$$

4. SOME IMPORTANT CURVES WHOSE KNOWLEDGE IS REQUIRED FOR THIS CHAPTER

4.1. Lines

(i) $x = 0$: Equation of y-axis
$y = 0$: Equation of x-axis

(ii) $x = a$: Equation of a line parallel to y-axis
$y = b$: Equation of a line parallel to x-axis

(iii) $y = x$: Equation of the line dividing quadrant I and quadrant III into two equal parts

(iv) $\frac{x}{a} + \frac{y}{b} = 1$: Equation of a line having intercepts a and b on axes.

(v) $y = mx$: Equation of a line with slope m and passing through the origin.

4.2. Modulus Functions

(i) $y = |x|$

(ii) $y = |x + 2|$

4.3 $y = x^3$: Cubic Function

4.4 $y = \frac{1}{x}$: Reciprocal Function

4.5 Circles

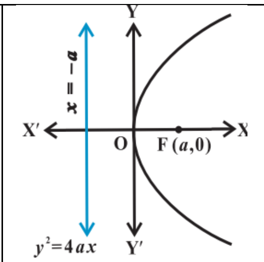
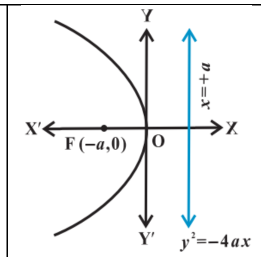
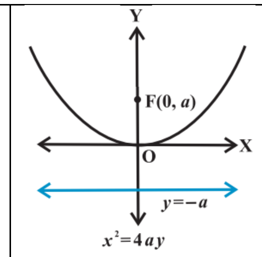
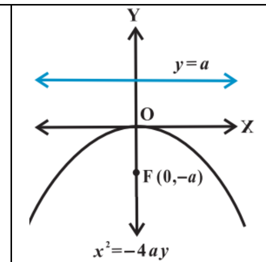
(i) $x^2 + y^2 = a^2$: Circle with centre at origin and radius a

(ii) $(x - a)^2 + y^2 = a^2$: Circle passing through origin, having centre on x-axis and radius a

(iii) $x^2 + (y - a)^2 = a^2$: Circle passing through origin, having centre on y-axis and radius a

(iv) $(x - a)^2 + (y - a)^2 = a^2$: Circle with centre at (a, a) and radius a **OR** Circle with radius a, lying in the first quadrant and touching the axes.

4.6. Parabola with length of latus rectum $4a$:

Figure				
Equation	$y^2 = 4ax$	$y^2 = -4ax$	$x^2 = 4ay$	$x^2 = 4ay$
Focus	$\mathrm{F}(a, 0)$	$\mathrm{F}(-a, 0)$	$\mathrm{F}(0, a)$	$\mathrm{F}(0, -a)$

4.7 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: Ellipse with lengths of axes as $2a$ and $2b$:

4.8 sine function : $y = \sin x$

4.9 cosine function : $y = \cos x$