FORMULAE & KEY POINTS

CLASS 12 MATHEMATICS

CHAPTER 04 : DETERMINANTS

1. DETERMINANT

To every **square matrix** A $= [a_{ij}]$ of order n, we associate a number (real or complex) called determinant of the square matrix A.

It is denoted by |**A**| or **det A** or **Δ**.

REMARKS

(i) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then determinant of A is written as $|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

(ii) For a matrix A, |A| is read as determinant of A and not modulus of A.

(iii) Only square matrices have determinants

2. RULES TO FIND THE DETERMINANT OF A MATRIX

(i) Determinant of a Square Matrix of Order One

Let $A = [a]$ be a square matrix of order 1, then $|A| = |a| = a$

Example

Let $A = [-\sqrt{2}]$, then $|A| = |-\sqrt{2}| = -\sqrt{2}$

(ii) Determinant of a Square Matrix of Order Two

Let $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ be a square matrix of order 2, then

$$|A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \times a_{22} - a_{21} \times a_{12}$$

Example

Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 5 \end{bmatrix}$, then $|A| = \begin{vmatrix} 2 & -3 \\ 4 & 5 \end{vmatrix} = 2 \times 5 - 4 \times (-3) = 10 + 12 = 22$

(iii) Determinant of a Square Matrix of Order Three

Let $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ be a square matrix of order 3, then |A| can be obtained by expanding it along any of the three rows R_1, R_2, R_3 or along any of the three columns C_1, C_2, C_3 in the following way:

Expanding the determinant along R_1 we get $|A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$|A| = [(-1)^{1+1} a_{11} \times$(det. obtained by deleting the row and column containing a_{11})]

$+[(-1)^{1+2} a_{12} \times$(det. obtained by deleting the row and column containing a_{12})]

$+[(-1)^{1+3} a_{13} \times$(det. obtained by deleting the row and column containing a_{13})]

$$= (-1)^{1+1} a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + (-1)^{1+2} a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + (-1)^{1+3} a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22}.a_{33} - a_{32}.a_{33}) - a_{12}(a_{21}.a_{33} - a_{31}.a_{23}) + a_{13}(a_{21}.a_{32} - a_{31}.a_{22})$$

Example

Let $A = \begin{bmatrix} 2 & -4 & 3 \\ 4 & 1 & -3 \\ 2 & 5 & 6 \end{bmatrix}$, then $|A| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 1 & -3 \\ 2 & 5 & 6 \end{vmatrix}$ and expanding it along R_1 we get,

$$|A| = (-1)^{1+1}.2 \begin{vmatrix} 1 & -3 \\ 5 & 6 \end{vmatrix} + (-1)^{1+2}.(-4) \begin{vmatrix} 4 & -3 \\ 2 & 6 \end{vmatrix} + (-1)^{1+3}.3 \begin{vmatrix} 4 & 1 \\ 2 & 5 \end{vmatrix}$$

$$= 2(6+15) + 4(24+6) + 3(20-2) = 2 \times 21 + 4 \times 30 + 3 \times 18 = 42 + 72 + 54$$

$$= 168$$

REMARKS

(i) The value of $|A|$ can be obtained by expanding it along any row or column.

(ii) For easier calculations, we expand the determinant along that row or column which contains maximum number of zeros

(iii) While expanding a determinant, instead of multiplying a_{ij} by $(-1)^{i+j}$, we can multiply a_{ij} by $(+1)$ or (-1) according as $(i+j)$ is even or odd. Thus to find the value of determinant of order 3 the elements circled in the following determinant are multiplied by (-1) :

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(iv) $|kA| = k^n |A|$, where n is the order of $|A|$.

(v) $|I_n| = 1$ where I_n is an identity matrix of order n

(vi) Let A be a diagonal matrix OR an upper triangular matrix OR a lower triangular matrix, then |A| = (product of the diagonal elements of A)

Example 1

Let $A = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$, then $|A| = \begin{vmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{vmatrix} = (-2)(3)(6) = -36$

Example 2

Let $B = \begin{bmatrix} -4 & 2 & -3 \\ 0 & -2 & 1 \\ 0 & 0 & 5 \end{bmatrix}$, then $|B| = \begin{vmatrix} -4 & 2 & -3 \\ 0 & -2 & 1 \\ 0 & 0 & 5 \end{vmatrix} = (-4)(-2)(5) = 40$

3. AREA OF A TRIANGLE

The area of a triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$, is given as

$$\Delta = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)] = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}.$$

REMARKS

(i) Since area is a positive quantity, we always take the absolute value of the above determinant.

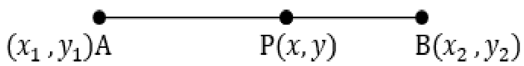
(Note that the actual value of Δ comes out to be negative if the points A, B, C are taken in clockwise direction)

(ii) If area is given, say ***k*** units, we use, both positive and negative values of the determinant for calculation. That is, we take $\frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \pm k$

(iii) The three points A, B, C are colllinear $\Leftrightarrow$ Area of the ΔABC is zero $\Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$

(iv) The equation of the line passing through the points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given as [*(Using the fact that if $P(x, y)$ is a points on the line AB then the points A, B and C are collinear]*

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$



4. MINOR (M_{ij}) AND COFACTOR (A_{ij}) OF AN ELEMENT a_{ij} OF A DETERMINANS

(i) MINOR (M_{ij})

For a given determinat $|A|$, the Minor, M_{ij} of an element a_{ij} is the determinant obtained by deleting i^{th} row and j^{th} column of $|A|$.

(i) COFACTOR (A_{ij})

Cofactor of the element a_{ij} of a determinant $|A|$ is defined as

$A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor of the element a_{ij}.

REMARKS

(i) Minor (or Cofactor) of an element of a determinant of order $n \geq 2$ is a determinant of order $(n - 1)$.

(ii) $A_{ij} = (-1)^{i+j}(M_{ij}) = \begin{cases} M_{ij}, & \text{if } (i+j) \text{is even} \\ -M_{ij}, & \text{if } (i+j) \text{is odd} \end{cases}$

(iii) $|A|$ = Sum of the product of the elements of any row (or column) with their corresponding cofactors.

Thus, $|A| = a_{11}.A_{11} + a_{12}.A_{12} + a_{13}A_{13}$ (Expanding along R_1)

Also, $|A| = a_{21}.A_{21} + a_{22}.A_{22} + a_{23}A_{23}$ (Expanding along C_2) and so on.

Example

Let, $|A| = \begin{vmatrix} 3 & -2 & 6 \\ -4 & 2 & -3 \\ 4 & 5 & 7 \end{vmatrix}$

Expanding along R_1 we get, $|A| = 3\begin{vmatrix} 2 & -3 \\ 5 & 7 \end{vmatrix} - (-2)\begin{vmatrix} -4 & -3 \\ 4 & 7 \end{vmatrix} + 6\begin{vmatrix} -4 & 2 \\ 4 & 5 \end{vmatrix}$

$= 3 \times 29 + 2 \times (-16) + 6 \times (-28) = 87 - 32 - 168 = -113$

Expanding along C_2 we get,

$|A| = -(-2)\begin{vmatrix} -4 & -3 \\ 4 & 7 \end{vmatrix} + 2\begin{vmatrix} 3 & 6 \\ 4 & 7 \end{vmatrix} - 5\begin{vmatrix} 3 & 6 \\ -4 & -3 \end{vmatrix}$

$= 2 \times (-16) + 2 \times (-3) - 5 \times 15 = -32 - 6 - 75 = -113$

We see that the value of |A| is independent of the choice of the row or column along which |A| is expanded.

REMARK

If elements of a row (or column) are multiplied with the cofactors of any other row (or column), then their sum is zero.

Thus, $a_{11}.A_{21} + a_{12}.A_{22} + a_{13}A_{23} = 0$

(Sum of the product of the elements of R_1 and corresponding cofactors of R_2)

Similarly, $a_{21}.A_{31} + a_{12}.A_{32} + a_{23}A_{33} = 0$

(Sum of the product of the elements of C_2 and corresponding cofactors of C_3.)

5. ADJOINT OF A MATRIX (*adj* A)

Let $A = [a_{ij}]_{n \times n}$ be a square matrix , then the adjoint of A denoted as

$adj\ A$ = Transpose of the Cofactor Matrix of A $= [A_{ij}]'_{n \times n} = [A_{ji}]_{n \times n}$

$$\Rightarrow adj\ A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}' = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

Examples

(i) Let, $A = \begin{bmatrix} -2 & -5 \\ 1 & 3 \end{bmatrix} \Rightarrow$ Cofactor matrix of $A = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}$

$$\Rightarrow adj\ A = \begin{bmatrix} 3 & -1 \\ 5 & -2 \end{bmatrix}' = \begin{bmatrix} 3 & 5 \\ -1 & -2 \end{bmatrix}$$

(ii) Let $B = \begin{bmatrix} 3 & -2 & 3 \\ 2 & 1 & -1 \\ 4 & -3 & 2 \end{bmatrix} \Rightarrow$ Cofactor matrix of $A = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}$

$$\Rightarrow adj\ A = \begin{bmatrix} -1 & -8 & -10 \\ -5 & -6 & 1 \\ -1 & 9 & 7 \end{bmatrix}' = \begin{bmatrix} -1 & -5 & -1 \\ -8 & -6 & 9 \\ -10 & 1 & 7 \end{bmatrix}$$

TRICK to find the Adjoint of a Square Matrix of Order 2

$$adj\ \mathrm{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Change Sign   Interchange

REMARKS

(i) $|\mathrm{AB}| = |\mathrm{A}||\mathrm{B}|$

(ii) $|\mathrm{A}^n| = |\mathrm{A}|^n$

(iii) $\mathrm{A}\,(adj\ \mathrm{A}) = (adj\ \mathrm{A})\,\mathrm{A} = |\mathrm{A}|\ \mathrm{I}$

Thus, for a square matrix A of order 3,

$$\mathrm{A}\,(adj\ \mathrm{A}) = (adj\ \mathrm{A})\,\mathrm{A} = |\mathrm{A}|\ \mathrm{I}_3 = \begin{bmatrix} |\mathrm{A}| & 0 & 0 \\ 0 & |\mathrm{A}| & 0 \\ 0 & 0 & |\mathrm{A}| \end{bmatrix}$$

(iv) From remark (iii) we get $|adj\ \mathrm{A}| = |\mathrm{A}|^{n-1}$

6. SINGULAR AND NON-SINGULAR MATRICES

(i) SINGULAR MATRICES

A square matrix A is said to be singular if $|\mathrm{A}| = 0$

Example

Let $\mathrm{A} = \begin{bmatrix} 3 & -1 \\ -6 & 2 \end{bmatrix}$, then $|\mathrm{A}| = 0$, hence A is a singular matrix.

(ii) NON-SINGULAR MATRICES

A square matrix A is said to be non-singular if $|\mathrm{A}| \neq 0$

Example

Let $\mathrm{B} = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$, then $|\mathrm{B}| = 2 \neq$, hence B is a non − singular matrix.

REMARKS

(i) If A and B are non − singular matrices of the same order, then AB and BA are also non − singular matrices of the same order

(ii) A is non-singular matrix $\Leftrightarrow$ A square matrix A is invertible $\Leftrightarrow \mathrm{A}^{-1}$ exists

(iii) (Inverse of a Non-singular Square Matrix A) $= \mathrm{A}^{-1} = \frac{1}{|\mathrm{A}|} \times (adj\ \mathrm{A})$

(iv) For two square matrices of order n each,

$$\mathrm{AB} = k\mathrm{I} \Leftrightarrow \mathrm{A}^{-1} = \left(\frac{1}{k}\right) \times \mathrm{B} \Leftrightarrow \mathrm{B}^{-1} = \left(\frac{1}{k}\right) \times \mathrm{A}$$

(v) $|\mathrm{A}^{-1}| = |\mathrm{A}|^{-1}$

(vi) $(\mathrm{A}^{-1})' = (\mathrm{A}')^{-1}$

(vii) $(\mathrm{AB})^{-1} = \mathrm{B}^{-1}\,\mathrm{A}^{-1}$ **(Reversal Law)**

7. CONSISTENT AND INCONSISTENT SYSTEM OF EQUATIONS

(i) Consistent System of Equations: A system of equations is said to be consistent if its solution (one or more) exists. A consistent system of equations has either a unique solution or an infinite number of solutions.

(ii) Inconsistent System of Equations: A system of equations is said to be inconsistent if its solution does not exist

8. SOLUTION OF SYSTEM OF LINEAR EQUATIONS USING INVERSE OF A MATRIX

Consider the system of equations

$$a_1x + b_1y + c_1 = d_1$$
$$a_2x + b_2y + c_2 = d_2$$
$$a_3x + b_3y + c_3 = d_3$$

Let $A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$

Then, the system of equations can be written as, AX = B, i.e.,

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

CASE I :

If A is non-singular then its inverse exists. The system has a unique solution given by

$$X = A^{-1}B$$

Case II :

If A is a singular matrix, then |A| = 0 and A^{-1} does not exist. In this case, we calculate (adj A) B.

(i) If (adj A) B ≠ O, then the system of equations is inconsistent.

(ii) If (adj A) B = O, then system may be either consistent or inconsistent according as the system has either infinitely many solutions or no solution.

9. SOLUTION OF SYSTEM OF LINEAR EQUATIONS HAVING UNIQUE SOLUTION USING INVERSE OF A MATRIX

Example

Solve the following system of equations using matrix method :

$x + y + z = 6$; $x + 2z = 7$; $3x + y + z = 12$

STEPS OF SOLUTION

Step I: The given system of equations can be written in matrix form as

$$AX = B \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 2 \\ 3 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} \qquad (1)$$

Step II: Now, $|A| = 1(0-2) - (1-6) + 1(1-0) = 4 \neq 0$

$\Rightarrow$ A is non-singular $\Rightarrow A^{-1}$exists. $\therefore$ from (1)

$$X = A^{-1}B \qquad (2)$$

Step III: Now, cofactor of the elements of matrix A are

$A_{11} = (0-2) = -2 \qquad A_{12} = -(1-6) = 5 \qquad A_{13} = -(1-3) = 2$

$A_{21} = -(1-1) = 0 \qquad A_{22} = (1-3) = -3 \qquad A_{23} = -(18-18) = 0$

$A_{31} = (2-0) = 2 \qquad A_{32} = -(2-1) = -1 \qquad A_{33} = (0-1) = -1$

Hence, $A^{-1} = \dfrac{adjA}{|A|} = \dfrac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -3 & -1 \\ 2 & 0 & - \end{bmatrix}$

Step IV: Hence, from (2)

$$X = A^{-1}B = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -3 & -1 \\ 2 & 0 & -1 \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 12 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow x = 3, y = 1, z = 2$

VERY IMPORTANT REMARK

Above type is a 'Sure Shot' question for Board Exam. But unfortunately many students lose marks in this by making one or the other mistake. So students are adviced to take care of the following preacautions:

(i) While calculating the cofactors, A_{12}, A_{21}, A_{23} and A_{32} put ' − ' sign in the beginning only as many students miss one or the other '−' sign.

(ii) Write the cofactors in the format given in the solution as it helps in writing A^{-1} conveniently.

(iii) Do not write A^{-1} as $\begin{bmatrix} -2/4 & 0 & 2/4 \\ 5/4 & -3/4 & -1/4 \\ 2/4 & 0 & -1/4 \end{bmatrix}$ by taking 1/4 inside the matrix as it makes the solution complicated.

CARELESS MISTAKE

Mistake: $X = BA^{-1} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}\dfrac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -3 & -1 \\ 2 & 0 & - \end{bmatrix}$

Correction: X = $X = A^{-1}B = \frac{1}{4}\begin{bmatrix} -2 & 0 & 2 \\ 5 & -3 & -1 \\ 2 & 0 & - \end{bmatrix}\begin{bmatrix} 6 \\ 7 \\ 12 \end{bmatrix}$

(*As matrix multiplication is non-commutative*, $A^{-1}B \neq BA^{-1}$)

MOST IMPORTANT QUESTIONS BASED ON FINDING SOLUTIONS OF A SYSTEM OF

LINEAR EQUATIONS

QUESTION 1

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear Equations : $x + 2y + z = 4;\ -x + y + z = 0; x - 3y + z = 2$

Hint : Here, Coefficient matrix = A' ∴ $X = (A')^{-1}B = (A^{-1})'B$.

Ans. $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$; $x = 9/5, y = 2/5;\ z = 7/5$

QUESTION 2

Find the product of $A = \begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$ and hence solve the system of linear equations : $x - y + z = 4;\ \ x - 2y - 2z = 9;\ 2x + y + 3z = 1$

Hint for Solution : Here, Coefficient matrix = B and $AB = 8I_3 \Rightarrow \left(\frac{1}{8}A\right)B = I_3$

$\Rightarrow B^{-1} = A$. Here, no need to find B^{-1} by $B^{-1} = \frac{\text{adj } B}{|B|}$

Ans. $AB = 8\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$; $x = 3, y = -2, z = -1$

QUESTION 3

Solve the following system of equations by matrix method:

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4;\quad \frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1;\quad \frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Hint for Solution : Here, take $X = \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix}$ so that $X = A^{-1}B$

$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} \Rightarrow \begin{bmatrix} 1/x \\ 1/y \\ 1/z \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/3 \\ 1/5 \end{bmatrix}$. Hence, $x = 2, y = 3, z = 5$

QUESTION 4

Use product $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$ to solve the following system of equations

$3x - 2y + 4z = 2;\ 2y - 3z = 1;\ \ x - y + 2z = 1$

	OR
	If $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$, find A^{-1} and hence solve the following system of linear Equations: $3x - 2y + 4z = 2;\ 2y - 3z = 1;\ x - y + 2z = 1$
	Hint for Solution : Here, note that the ordere of the equations in the given system has tro be changed in the following way so that the first matrix becomes the Coefficient Matrix for the system: **Given Order** : $3x - 2y + 4z = 2;\ 2y - 3z = 1;\ x - y + 2z = 1$ **Changed Order :** $x - y + 2z = 1;\ 2y - 3z = 1;\ 3x - 2y + 4z = 2$ Now proceed as in Example 2. **Ans.** $x = 0, y = 5, z = 3$