FORMULAE & KEY POINTS
CLASS 12 MATHEMATICS
CHAPTER 03 : MATRICES

1. MATRICES

1.1 DEFINITION OF MATRIX

A matrix is an ordered rectangular array of numbers or functions in the form c Rows and n Columns.

The numbers or functions belonging to a matrix are called the Elements or the Entries of the matrix.

1.2 ORDER OF A MATRIX

A matrix having m rows and n columns is called a matrix of order $m \times n$ or simply $m \times n$ matrix (read as an m by n matrix)

TRICK TO REMEMBER

Generally, students get confused that while writing order of a matrix we should write number of rows first or number of columns first. As a trick, let in the spelling 'ORDER', the first letter O stand for Order, and the second letter R stands for Rows. Thus, while writing order of a matrix we first write Number of Rows and then Number of Columns.

1.3 A matrix is denoted by capital letter like A, B, C, X, Y, P, Q etc. and the elements of a matrix are denoted by small letters a, b, c, x, y, z, p, q etc.

1.4 A general matrix of order $m \times n$ can be taken as

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1j} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2j} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3j} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{i1} & a_{i2} & a_{i3} & \dots & a_{ij} & \dots & a_{in} \\ \vdots & \vdots & \vdots & & \vdots & & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \dots & a_{mj} & \dots & a_{mn} \end{bmatrix}_{m \times n}$$

OR

$A = [a_{ij}]_{m \times n}, 1 \le i \le m; 1 \le j \le n;\ i, j \in N.$

The element a_{ij} is called *the* $(i, j)^{th}$ element of the matrix A

1.5 Examples

(i) $A = \begin{bmatrix} 2 & \sqrt{3} & -5 \\ \frac{2}{3} & 0 & -1 \end{bmatrix}_{2\times 3}$ (ii) $B = \begin{bmatrix} 3-i & 4 & -\frac{2}{3} \\ 4.5 & -2 & 6 \\ 1 & 2 & \sqrt{5} \end{bmatrix}_{3\times 3}$

(iii) $C = \begin{bmatrix} 1+y & y^3 \\ \cos x & (\sin x - 2) \\ \tan x & \sqrt{3} \end{bmatrix}_{3\times 2}$

2. TYPES OF MATRICES

2.1 ROW MATRIX

A matrix is said to be a row matrix if it has only one row.

Examples

$A = [x \quad -y]_{1\times 2}$; $B = [2 \quad \sqrt{3} \quad -1]_{1\times 3}$

REMARK

The order of a row matrix is of he form $1 \times n$.

2.2 COLUMN MATRIX

A matrix is said to be a column matrix if it has only one column.

Examples

$A = \begin{bmatrix} 3 \\ -2 \end{bmatrix}_{2\times 1}$, $B = \begin{bmatrix} \sqrt{2} \\ x \\ -1 \end{bmatrix}_{3\times 1}$

REMARK

The order of a row matrix is of he form $m \times 1$.

2.3 ZERO MATRIX OR NULL MATRIX (O)

A matrix is said to be zero matrix or null matrix, denoted as O if all its elements zero. Symbolically,

$A = [a_{ij}]_{m \times n}$ is a zero matrix if $a_{ij} = 0$, for every i, j.

Examples

$[0]$, $[0 \quad 0]$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

2.4 SQUARE MATRIX

A matrix in which the number of rows is equal to the number of columns, is said to be a square matrix.

Examples

$P = \begin{bmatrix} 5 & 1 \\ -2 & \sqrt{3} \end{bmatrix}_{2\times 2}$, $Q = \begin{bmatrix} 2 & -1 & \sqrt{5} \\ 3 & 0 & -4 \\ 6 & -2 & 1 \end{bmatrix}_{3\times 3}$

REMARKS

(i) A square matrix of order $n \times n$ is termed as a square matrix of order n.

(ii) If A $= [a_{ij}]$ is a square matrix of order n, then the elements (entries) $a_{11}, a_{22}, \dots, a_{nn}$ are said to constitute the diagonal of the mtrix A.

Thus, in the above examples, the diagonal elements of P are 5, $\sqrt{3}$ and the diagonal elements of Q are 2, 0, −1

2.5 DIAGONAL MATRIX

A square matrix B $= [b_{ij}]_{m \times m}$ is said to be a diagonal matrix if all its non-diagonal elements are zero. Symbolically,

A matrix B $= [b_{ij}]_{m \times n}$ is a diagonal matrix $\Leftrightarrow \begin{cases} m = n, \text{that is, B is a square matrix} \\ b_{ij} = 0, \quad \text{if } i \neq j \end{cases}$

Examples

$$A = [3], \quad B = \begin{bmatrix} -4 & 0 \\ 0 & 3 \end{bmatrix}_{2\times2}, \quad C = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}_{3\times3}$$

are diagonal matrices of order 1, 2, 3 respectively

REMARK

A square zero matrix is also a diagonal matrix.

2.6 SCALAR MATRIX

A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal. Symbolically,

A matrix B $= [b_{ij}]_{m \times n}$ is a scalar matrix $\Leftrightarrow \begin{cases} m = n, \text{that is, B is a square matrix} \\ b_{ij} = 0, \quad \text{if } i \neq j \\ b_{ij} = k, \ \text{if } i = j, \text{for some constant } k \end{cases}$

Examples

$$A = [4], \quad B = \begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{3} \end{bmatrix}_{2\times2}, \quad C = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}_{3\times3}$$

are scaler matrices matrices of order 1, 2, 3, respectively

REMARK

A square zero matrix is also a scaler matrix.

2.7 IDENTITY MATRIX (I_n)

A square matrix in which each diagonal element is 1 and all other elements are zero is called an identity matrix. Symbolically,

A matrix A $= [a_{ij}]_{m \times n}$ is an identity matrix $\Leftrightarrow \begin{cases} m = n \text{ that is, A is a square matrix} \\ a_{ij} = 0, \quad \text{if } i \neq j \\ a_{ij} = 1, \quad \text{if } i = j \end{cases}$

Examples

$$I_1 = [1]_{1\times1}, \quad I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times2}, \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times3}$$

are scaler matrices matrices of order 1, 2, 3, respectively.

REMARKS

(i) An identity matrix is called so because for a square matrix A and an identity matrix I, each of order n, $A \times I = A = I \times A$

(ii) An identity matrix of order 1, 2, 3 etc. is denoted as I_1, I_2, I_3 etc. respectively.

2.8 UPPER TRIANGULAR MATRIX

An upper triangular matrix is a square matrix in which all the entries below the main diagonal are zero. Symbolically,

A matrix $A = [a_{ij}]_{m \times n}$ is an upper triangular matrix

$$\Leftrightarrow \begin{cases} m = n \text{ that is, B is a square matrix} \\ a_{ij} = 0, \qquad \text{if } i > j \end{cases}$$

Examples

(i) $A = \begin{bmatrix} -2 & 1 \\ 0 & 3 \end{bmatrix}$ **(ii)** $B = \begin{bmatrix} 3 & -3 & 2 \\ 0 & -\sqrt{5} & -1 \\ 0 & 0 & 4 \end{bmatrix}$

REMARK

In an upper triangular matrix, elements on the main diagonal or above the main diagonal can also be zero.

2.9 LOWER TRIANGULAR MATRIX

A lower triangular matrix is a square matrix in which all the entries above the main diagonal are zero. Symbolically,

A matrix $B = [b_{ij}]_{m \times n}$ is a lower triangular matrix

$$\Leftrightarrow \begin{cases} m = n \text{ that is, B is a square matrix} \\ b_{ij} = 0, \qquad \text{if } i < j \end{cases}$$

Examples

(i) $A = \begin{bmatrix} -4 & 0 \\ 2 & 5 \end{bmatrix}$ **(ii)** $B = \begin{bmatrix} -2 & 0 & 0 \\ 8 & -\sqrt{3} & 0 \\ 1 & 2 & 6 \end{bmatrix}$

REMARK

In a lower triangular matrix, elements on the main diagonal or below the main diagonal can also be zero.

3. EQUALITY OF MATRICES

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if

(i) Order of A = Order of B

(ii) Each element of A is equal to the corresponding element of B, that is $a_{ij} = b_{ij}$ for all i and j

Examples

(i) $\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} x & y & z \\ a & b & c \end{bmatrix} = \begin{bmatrix} -1 & 0 & \sqrt{3} \\ 2.4 & 2/3 & 4 \end{bmatrix} \Rightarrow x = -1, y = 0, z = \sqrt{3}, a = 2.4, b = 2/3, c = 4$

4. OPERATIONS ON MATRICES

4.1 ADDITION OF MATRICES

4.1.1 DEFINITION

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are two matrices of the **Same Order**, say $m \times n$.
Then, the sum of A and B is defined as matrix, $C = A + B = [c_{ij}]_{m \times n}$,
where $c_{ij} = a_{ij} + b_{ij}$, for all possible values of i and j.

Example

Let, $A = \begin{bmatrix} 2 & \sqrt{3} & -5 \\ \frac{2}{3} & 0 & -1 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} 3 & 2 & 1 \\ 5 & 3 & \frac{1}{2} \end{bmatrix}_{2\times3}$ then

$$A + B = \begin{bmatrix} 2+3 & 2+\sqrt{3} & -5+1 \\ \frac{2}{3}+5 & 0+3 & -1+\frac{1}{2} \end{bmatrix}_{2\times3} = \begin{bmatrix} 5 & 2+\sqrt{3} & -4 \\ \frac{17}{3} & 3 & -\frac{1}{2} \end{bmatrix}_{2\times3}$$

REMARK

If A and B are not of the same order, then $A + B$ is not defined.

4.1.2 PROPERTIES OF MATRIX ADDITION

(i) Commutative Law

$A + B = B + A$

(ii) Associative Law

$(A + B) + C = A + (B + C)$.

(iii) Existence of additive identity

For every matrix A of order $m \times n$, there exists a zero matrix O of order $m \times n$ such that $A + O = O + A = A$. Here, O is called the additive identity for matrix addition.

(iv) Existence of additive inverse

For every matrix A of order $m \times n$, there exists a matrix $(-A)$ of order $m \times n$ such that $A + (-A) = O = (-A) + A$. Here, $(-A)$ is called the additive inverse of the matrix A and vice-versa.

REMARK

$(-A)$ is called the additive inverse of $A \Leftrightarrow A$ is the additive inverse of $(-A)$

4.2 DIFFERENCE OF MATRICES

If A $= [a_{ij}]$ and B $= [b_{ij}]$ are two matrices of the **Same Order**, say $m \times n$, then, the difference of A and B is defined as matrix, D $=$ A $-$ B $= [d_{ij}]_{m \times n}$, where $d_{ij} = a_{ij} - b_{ij}$, for all possible values of i and j.

Example

Let, $A = \begin{bmatrix} 4 & \sqrt{3} & -5 \\ \frac{2}{3} & 0 & -1 \end{bmatrix}_{2\times3}$, $B = \begin{bmatrix} 3 & 2 & 1 \\ 5 & -3 & \frac{1}{2} \end{bmatrix}_{2\times3}$ then

$$A + B = \begin{bmatrix} 4-3 & \sqrt{3}-2 & -5-1 \\ \frac{2}{3}-5 & 0-3 & -1-\frac{1}{2} \end{bmatrix}_{2\times3} = \begin{bmatrix} 1 & \sqrt{3}-2 & -6 \\ -\frac{13}{3} & 3 & -\frac{3}{2} \end{bmatrix}_{2\times3}$$

REMARK

If A and B are not of the same order, then A $-$ B is not defined.

4.3 MULTIPLICATION OF A MATRIX BY A SCALAR

4.3.1 DEFINITION

If A $= [a_{ij}]_{m \times n}$ is a matrix and k is a scalar, then kA is another matrix which is obtained by multiplying each element of A by the scalar k. Symbolically,

for a matrix A $= [a_{ij}]_{m \times n}$, $kA = k\,[a_{ij}]_{m \times n} = [k\,(a_{ij})]_{m \times n}$

Example

Let $A = \begin{bmatrix} -1 & 2 & 4 \\ \sqrt{3} & 3 & 5 \end{bmatrix}_{2\times3} \Rightarrow 2A = \begin{bmatrix} -2 & 4 & 8 \\ 2\sqrt{3} & 6 & 10 \end{bmatrix}_{2\times3}$

REMARK

For a matrix A of order $m \times n$, the matrix $(-1)A = -A$ is called the negative of the the matrix A.

4.3.2 PROPERTIES OF SCALAR MULTIPLICATION OF A MATRIX

If A $= [a_{ij}]$ and B $= [b_{ij}]$ are two matrices of the same order, say $m \times n$, and k and l are scalars, then

(i) $k(A + B) = kA + kB$ (ii) $(k + l)A = kA + lA$

4.4 MULTIPLICATION OF TWO MATRICES (A × B)

4.4.1 Let A $= [a_{ij}]_{m\times n}$ be a matrix of order $m \times n$ and B $= [b_{ij}]_{p\times q}$ be a matrix of order $p \times q$, then we have the following results:

(i) The product A × B is defined only when

(The number of columns of A) = (The number of rows of B) $i.e.$ $\boldsymbol{n = p}$

(ii) Under the condition $n = p$, the order of the product matrix, say C, is given as

$m \times p$ $i.e.$ (Number of rows in A) × (Number of Columns in B).

(iii) The elements c_{ij} of the product matrix C $= [c_{ij}]_{m\times p}$ are given as

c_{ij} = Sum of the product of the corresponding elements of the i^{th} row of the first matrix A and the j^{th} column of the second matrix B.

(iv) From (iii) above, since i^{th} row of A is $[a_{i1} \quad a_{12} \quad \cdots \quad a_{in}]$ and j^{th}column of B is $\begin{bmatrix} b_{1k} \\ b_{2k} \\ \vdots \\ b_{nk} \end{bmatrix}$

then then $c_{ik} = a_{i1}\, b_{1k} + a_{i2}\, b_{2k} + a_{i3}\, b_{3k} + \ldots + a_{in}\, b_{nk} = \sum_{j=1}^{n} a_{ij}\, b_{jk}$

Example

Let, $A = \begin{bmatrix} 1 & -3 \\ 5 & 2 \\ -3 & 6 \end{bmatrix}_{3\times 2}$ and $B = \begin{bmatrix} 4 & 3 & -5 \\ 2 & 7 & -1 \end{bmatrix}_{2\times 3}$, then

$$AB = \begin{bmatrix} c_{11} & c_{12} & ac_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}_{3\times 3}$$

$$= \begin{bmatrix} 1\times 4 + (-3)\times 2 & 1\times 3 + (-3)\times 7 & 1\times(-5) + (-3)\times(-1) \\ 5\times 4 + 2\times 2 & 5\times 3 + 2\times 7 & 5\times(-5) + 2\times(-1) \\ (-3)\times 4 + 6\times 2 & (-3)\times 3 + 6\times 7 & (-3)\times(-5) + 6\times(-1) \end{bmatrix}$$

$$= \begin{bmatrix} -2 & -18 & -2 \\ 24 & 29 & -27 \\ 0 & 33 & 9 \end{bmatrix}$$

whereas, $BA = \begin{bmatrix} 4 & 3 & -5 \\ 2 & 7 & -1 \end{bmatrix}_{2\times 3} \begin{bmatrix} 1 & -3 \\ 5 & 2 \\ -3 & 6 \end{bmatrix}_{3\times 2} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}_{2\times 2}$

$$= \begin{bmatrix} 4\times 1 + 3\times 5 + (-5)\times(-3) & 4\times(-3) + 3\times 2 + (-5)\times 6 \\ 2\times 1 + 7\times 5 + (-1)\times(-3) & 2\times(-3) + 7\times 4 + (-1)\times 6 \end{bmatrix} = \begin{bmatrix} 34 & -36 \\ 40 & 16 \end{bmatrix}$$

PROPERTIES OF MATRIX MULTIPLICATION

(i) **If AB is defined, then BA may or may not be defined.**
For example, if A is of order 3×2 and B is of order 2×2 then AB is defined and is of order 3×2, but BA is not defined since (No. of columns of A) $\neq$ (No. of rows of B)

(ii) **Product of matrices is non-commutative ie. AB $\neq$ BA, in general.** But this does not mean that AB $\neq$ BA for every pair of matrices A and B for which AB and BA are defined. In fact, the multiplication of diagonal matrices of same order will be commutative. The following examples will make clear the above points:

Example 1 **(AB and BA both are defined but are of different order so AB $\neq$ BA)**

If A is of order 3×2 and B is of order 2×3 then
AB is defined and is of order 3×3;
BA is also defined but is of order 2×2.
Clearly, AB $\neq$ BA as two matrices of different orders can never be equal.

Example 2 **(AB and BA have same order but AB $\neq$ BA)**

Let $A = \begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 1 \\ -1 & 3 \end{bmatrix}$ then $AB = \begin{bmatrix} -1 & -7 \\ -9 & 7 \end{bmatrix}$ and $BA = \begin{bmatrix} 0 & 7 \\ 10 & 6 \end{bmatrix}$.

Clearly, the matrices AB and BA have same order but $AB \neq BA$

Example 3 (**AB and BA have same order and AB = BA**)

Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 0 \\ 0 & 3 \end{bmatrix}$ then $AB = \begin{bmatrix} -4 & 0 \\ 0 & 3 \end{bmatrix} = BA$

(iii) **AB and BA both are defined $\Leftrightarrow$ orders of A and B are reverse of each other**.
That is, if A is a matrix of order $m \times n$ and B is of order $n \times m$, then AB and BA both are defined.

(iv) **Zero Matrix as a Product of Two Non-zero Matrices**
If the product of two matrices is a zero matrix, it is not necessary that one of the matrices is a zero matrix. That is, $AB = O$ does not necessarily impliy either $A = 0$ or $B = 0$.
($\textit{Note that in case of real numbers}, ab = 0 \Rightarrow a = 0 \textit{ or } b = 0.$)

Example 1 (**$A \neq O, B \neq O$ and $AB = BA = O$**)

Let $A = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 3 \\ 0 & 0 \end{bmatrix}$ then $AB = BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Example 2 (**$A \neq O, B \neq O$ and $AB = O$ but $BA \neq O$**)

Let $A = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then $AB = O$ but $BA = \begin{bmatrix} 2 & -2 \\ 2 & -2 \end{bmatrix}$

(v) For any two square matrices of same order, $A = O$ or $B = O \Rightarrow AB = BA = O$

4.4.2 PROPERTIES OF MULTIPLICATION OF MATRICES

(i) **The Associative Law**: $(AB)C = A(BC)$,

(ii) **The Distributive Law**: (**i**) $A(B + C) = AB + AC$ (**ii**) $(A + B)C = AC + BC$

(iii) **The Existence of Multiplicative Identity**: For every square matrix A, there exists an identity matrix I of same order such that $IA = AI = A$.

5. TRANSPOSE OF A MATRIX

5.1 DEFINITION

If $A = [a_{ij}]$ be an $m \times n$ matrix, then the transpose of A, denoted as A' or A^T, is the matrix obtained by interchanging the rows and columns of A. Symbolically,

$$A = [a_{ij}]_{m \times n} \Rightarrow A' = [a_{ji}]_{n \times m}$$

5.2 PROPERTIES OF TRANSPOSE OF THE MATRICES

(i) $(A')' = A$ (ii) $(kA)' = kA'$ (where k is any constant)

(iii) $(A + B)' = A' + B'$ (iv) $(A - B)' = A' - B'$

(v) $(AB)' = B'A'$ (**Reversal Law**)

6. SYMMETRIC AND SKEW SYMMETRIC MATRICES

6.1 SYMMETRIC MATRIX

A square matrix $A = [a_{ij}]$ is said to be symmetric if $A' = A$

Thus, A is symmetric $\Leftrightarrow a_{ij} = a_{ji}$

Example

Let $A = \begin{bmatrix} -2 & 3 & 4 \\ 3 & 1 & -\sqrt{2} \\ 4 & -\sqrt{2} & 6 \end{bmatrix}$, then $A' = A$. Hence A is symmetric matrix.

6.2 SKEW SYMMETRIC MATRIX

A square matrix $A = [a_{ij}]$is said to be symmetric if $A' = -A$

Thus, A is symmetric $\Leftrightarrow a_{ij} = -a_{ji}$

Example

Let $A = \begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$, then $A' = \begin{bmatrix} 0 & -a & b \\ a & 0 & -c \\ -b & c & 0 \end{bmatrix} = -A.$

Hence A is skew symmetric matrix.

REMARKS

(i) All the diagonal elements of a skew symmetric matrix are zero.
Thus, A is skew symmetric matrix $\Leftrightarrow a_{ii} = 0$ for each i.

(ii) For a square matrix A, with real number entries,
$A + A'$ is a symmetric matrix and
$A - A'$ is a skew symmetric matrix

(iii) Any square matrix A can be expressed as the sum of a symmetric and a skew symmetric matrix as $A = P + Q$ where,
$P = \frac{1}{2}(A + A')$ is symmetic and $Q = \frac{1}{2}(A - A')$ is a skew symmetric matrix.

7. INVERTIBLE MATRIX

A square matrix A of order m is said to be invertible if there exists another square matrix B of the same order m, such that $AB = BA = I$.
In this case, B is called the inverse of matrix A and it is denoted by A^{-1}.

REMARKS

(i) A rectangular matrix does not possess inverse matrix

(ii) A is the Inverse of B $\Leftrightarrow$ B is the Inverse of A

(iii) Inverse of a square matrix, if it exists, is unique.

(iv) $(AB)^{-1} = B^{-1}A^{-1}$ **(Reversal Law)**

A VERY IMPORTANT QUESTION

Show that $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ is a zero of the polynomial $x^2 - 4x + 7$ (**or** a root of the quadratic equation $x^2 - 4x + 7 = 0$). Hence find

(i) A^{-1} **(ii)** A^5

SOLUTION

Substituting A in place of x in $x^2 - 4x + 7$ we get

$$A^2 - 4A + 7\mathbf{I} = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 4\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - \begin{bmatrix} 8 & 12 \\ -4 & 8 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \mathbf{O}$. Hence, A is a zero of the given polynomial

Careless Mistake: (i) *Students miss I and write* $A^2 - 4A + 7$ *and lose marks*

(ii) *Students write* 0 *in place of* **O** *and lose marks*

(i) Consider $A^2 - 4A + 7I = O$. Multiplying this equation by A^{-1} we get :

$A^2A^{-1} - 4AA^{-1} + 7IA^{-1} = OA^{-1} \Rightarrow A - 4I + 7A^{-1} = O$

$$\Rightarrow A^{-1} = \frac{1}{7}(4I - A) = \frac{1}{7}\left(4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}\right) = \frac{1}{7}\begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix}$$

(ii) Consider $A^2 - 4A + 7I = O \Rightarrow A^2 = 4A - 7I$ **(1)**

$A^5 = [(A^2)^2]A = [(4A - 7I)^2]A$ [using (1)]

$= [16A^2 - 56AI + 49I^2]A = [16(4A - 7I) - 56A + 49I]A$ [using (1)]

$= [64A - 112I - 56A + 49I]A = [8A - 63I]A = 8A^2 - 63AI$

$= 8(4A - 7I) - 63A$ [using (1)]

$$= 32A - 56I - 63A = -31A - 56I = -31\begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} - 56\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$$